Multiphase Approach to Thaw Subsidence of Unsaturated Frozen Soils: Equation Development

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Abstract: Construction and maintenance of engineering structures in arctic regions demands an understanding of and the ability to cope with environmental problems produced by permafrost. Where thawing of the permafrost cannot be prevented, the range, amount, and extent of degradation are factors that must be taken into account in the design of engineering structures. Predictions of these factors are facilitated by the use of a mathematical model designed to predict the physical system under consideration. This study addresses the problem by developing a predictive model to describe permafrost thaw subsidence based upon a complete formulation of the problem. This problem is one of multiphase (air, ice, water, and solid) transport and deformation in a porous medium. The conservation of mass equations for liquid water, ice with phase change, and deforming soil solids are developed. Darcy's law is extended to include moisture movement due to thermal gradients. The quasi-static equilibrium equations and stress-strain relations that assume a perfectly elastic solid matrix are also employed. Simultaneous variations of moisture retention and phase-composition curves with temperature and pore pressure are also incorporated in the model. The energy-conservation equation, which includes terms due to viscous dissipation and compression effects, along with the other equations expressed, is found to satisfactorily represent the physical processes occurring in thawing permafrost soils. Approximations to simulate the thawing of a one-dimensional soil column are then developed from the general model.

INTRODUCTION AND LITERATURE SURVEY

Permafrost is any rock or soil material that has remained below the freezing temperature of water continuously for 2 or more years. It underlies 20% of the earth's surface area and, when disturbed by various human activities, can cause many unique problems. Thawing of permafrost with subsequent surface subsidence under unheated structures (such as roads, airfields, and railroads) and ground subsidence under heated structures are two main types of permafrost-related problems. Thaw consolidation due to melting of ground ice is a costly problem. Annual maintenance, due to thaw consolidation, cost Alaskan railroads $87 for each unit length of rail in 1970 (Pewe 1982). Embankment thicknesses necessary to prevent thaw and differential settlement are often economically impractical. This is especially true in discontinuous permafrost areas in which permafrost temperatures approach 0°C.

Effective planning of construction for transportation and buildings above frozen ground requires the ability to predict the thaw behavior of permafrost under a variety of thermal conditions. Numerous researchers have spent much effort on the simulation of permafrost thaw consolidation and developed several models. Works by Tsytovich (1960) and Tsytovich et al. (1966) appear to be the first studies on this subject. These studies conclude that thaw consolidation caused by the abrupt change in void ratio that occurs during thawing of the soil ice is approximately equal to the thickness of ice inclusions, is independent of external load, is proportional to depth of thawing, and occurs at a rate proportional to the square root of time. For a considerable period after complete thawing has occurred, settlement proceeds at a reduced rate and is proportional to the logarithm of the time. Later, Tsytovich et al. (1965) showed that the settlement of thawing saturated soils obeys the theory of consolidation as formulated by Terzaghi.

A one-dimensional study of permafrost thaw and settlement was later investigated by various researchers. Some of them approached the problem from a practical point of view [e.g., Zaretskii (1968), Brown and Johnston (1970)], while others preferred a more theoretical approach. Morgenstern and Nixon (1971) formulated thawing settlement in terms of the theory of consolidation, and provided a solution to a moving-boundary thaw-consolidation problem in which the thaw line is assumed to move proportionally to the square root of time. One-dimensional closed-form solutions have been obtained for several cases of practical interest. It was shown that the excess pore pressures and the degree of consolidation in thawing soils depend primarily on the thaw-

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consolidation ratio, which is a function of the coefficient of consolidation. A similar approach was used by Ryden (1985). Nixon and Morgenstern (1973) later extended their study to incorporate arbitrary movements of the thaw line, including a nonlinear compressibility relation. McRoberts et al. (1978) considered certain aspects of thaw-consolidation effects in degrading permafrost by examining two cases. Temperature effects were introduced through a moving thaw line into Morgenstern and Nixon's formulation. Morgenstern and Nixon considered only saturated soils, and ice content in the medium was not taken into account. Their study also assumed no liquid water was present at temperatures below freezing, all of it having been converted to ice at the freezing point. This is not the case in unsaturated frozen soils, which always contain some unfrozen water. Yanagiswa and Yao (1985) produced a closed-form solution to a heave problem by using a similar method. They regarded the depth of frost penetration to be a linear function of the square root of time, and the resulting frost heave is related to the depth of frost penetration. Coulter (1983) took a similar approach by solving heat and flow equations independently. The solution of the heat equation provided the location of the thaw boundary, which defines the solution domain for the excess pore-water equation.

There are also various studies to simulate the coupled mass and heat transport in a freezing deformable soil system in addition to studies for rigid porous media. Models for rigid porous media do not make provisions for deformation of the soil matrix [e.g., Jame and Norum (1980), Taylor and Luthin (1978), Mohan (1975), Guymon and Luthin (1974), Harlan (1973), and Fuchs et al. (1978)]. A complete review of frost heave models has been given by O'Neill (1983). Studies on deformable media cover the works of Sheppard et al. (1978) and Del Guidice et al. (1978). Sheppard et al. (1978) developed a model that described heat and water flow in a freezing soil, and has provisions for incorporating the effect of overburdened pressure and deformation of the soil matrix. Rigid-ice models for frost heave have also been supplied by O'Neill and Miller (1985), Miller (1972, 1977, 1978), and Holden et al. (1985), which consider an ice lens growing behind the freezing front, thus formulating the rigid-ice model of secondary heave. Holden reformulated the equation supplied by Miller so that it gives rise to a pair of nonlinear ordinary differential equations, which are more readily solved by numerical techniques. Del Guidice et al. (1978) presented the finite-element model, formulated for nonlinear heat conduction, to study ground-freezing problems by solving the energy equation only. Sykes et al. (1974a,b) solved two-dimensional problems by the finite-element method to determine the effects of settlement and the significance of variable geometry and forced convection. Later, Sykes and Lennox (1976) considered a nonlinear stress-strain relation for thawing permafrost. Again, as in Sheppard et al.'s (1978) model, Sykes et al. considered only saturated soil in addition to neglecting the term for ice content. Civan and Sliepcevich (1985) used an enthalpy method to handle soil freezing and thawing. Here again, only the energy equation was employed. Water migration due to temperature gradient in frozen soils was experimentally verified by various researchers [e.g., Hoekstra (1966), Fuchs et al. (1978), Oliphant et al. (1983), Wood and Williams (1983), and Kung and Steenhuis (1986)]. Vignes and Dijkeme (1974) found the rate of water transport toward an ice front to be constant under isothermal conditions. Furthermore, in a series of papers, Nakano et al. (1983, 1984a,b) studied the effects of ice content on the transport of water in frozen soil. Perfect et al. (1991) reviewed heat and mass transfer models developed in the previous decade.

Bear and Corapcioglu (1981) developed a mathematical model for fluid pressure, temperature, and land subsidence owing to temperature and pressure changes in a saturated porous medium. Conservation of mass, energy, and equilibrium equations were developed, and the effects of viscous dissipation and compressible work have been included in the formulation. The effects of heat transfer by forced convection as well as conduction were included in the model.

One may summarize the limitations of all current models collectively by noting that none deal with mechanics of porous media of any complexity [e.g., consolidation, unsaturated medium, frozen moisture (ice) content] and none completely couple the governing equations (e.g., equilibrium equations, conservation of mass equations for frozen and unfrozen water, and the heat-balance equation). This study presents a mathematical model that includes coupled transport equations to obtain the governing equation of thaw of a partially frozen porous medium based on concepts developed by Panday and Corapcioglu (1991) and Corapcioglu et al. (1993). In Panday and Corapcioglu (1995), we present a numerical solution to the model and the sensitivity of the model to various model parameters.

GOVERNING EQUATIONS

Unfrozen-Water-Flow Equation

Thaw subsidence of partially frozen unsaturated soils is quite a complex phenomenon. At macroscopic level, there exist four distinct phases (soil solids, ice, water, and air) within any representative elementary volume of the soil system. Due to an increase of temperature, the ice within the soil system starts melting, thus creating a two-fold effect. First, due to thawing and associated drainage, the pore-water pressure decreases. This leads to an increase in effective stresses on the soil grains, thus causing deformations. Furthermore, thaw settlement occurs due to a volume change in the melting ice. The soil air is assumed to remain at atmospheric pressure, thus neglecting conservation of mass equation for air during the formulation. Fuchs et al. (1978) report that vapor phase contributes insignificantly to transport phenomena, thus justifying the assumption of atmospheric air pressure.

In this study, we employ a conventional continuum hypothesis by utilizing continuous variables to quantify the transport and amount of mass of each phase (Corapcioglu and Baehr 1987). The aqueous phase (water) is characterized by volumetric content θ_w, density ρ_f, and mass flux vector $(\mathbf{q}\rho_f)$. The solid phases, ice, and soil grains are assumed to be incompressible individually; however, the soil matrix as a whole deforms. Ice and soil grains are characterized by volumetric contents θ_i and $(1 - n)$ and densities ρ_i and ρ_s, respectively. By definition, the volumetric contents are related to the porosity as $\theta_i + \theta_a + \theta_w = n$. Changes in the volumetric contents of the phases due to mass transfer (e.g., evaporation, sublimation, condensation) other than melting and freezing are assumed to be negligible.

Then a macroscopic conservation of mass equation can be written for each phase in the medium. The three-dimensional conservation of mass equation for the unfrozen water phase is given by the following:

$$\nabla \cdot (\rho_f \mathbf{q}) + \frac{\partial}{\partial t}(\rho_f \theta_w) - R_a = 0 \tag{1}$$

For a deforming porous medium, the specific discharge of water relative to the moving solid is expressed by the following modified Darcy's law:

$$\mathbf{q}_r = \theta_w(\mathbf{V}_f - \mathbf{V}_s) = \mathbf{q} - \theta_w \mathbf{V}_s = -\mathbf{K}\left(\frac{1}{\rho_f g}\nabla p - \nabla z\right) - \mathbf{D}_{MT}\nabla T \tag{2}$$

where the flow is induced by potential as well as thermal gradients. The term with thermal gradient in (2) represents the movement due to temperature difference of unfrozen water into the frozen zone. Assuming that the solid grains are incompressible, i.e., ρ_s = constant, then the equation of solid-mass conservation can be expressed as follows:

$$\nabla \cdot [(1 - n)\mathbf{V}_s] + \frac{\partial}{\partial t}(1 - n) = 0 \tag{3}$$

Also, with $\mathbf{U}$ = solids displacement vector, and ε = volumetric strain, then by definition

$$\varepsilon = \nabla \cdot \mathbf{U} = \frac{\partial U_x}{\partial x} + \frac{\partial U_y}{\partial y} + \frac{\partial U_z}{\partial z} \tag{4}$$

Employing the concept of material derivatives with respect to moving solids and substituting (4) and (5) into (3), the strain rate can be expressed as follows:

$$\mathbf{V}_s = \frac{d_s\mathbf{U}}{dt}; \quad \frac{d_s\varepsilon}{dt} = \frac{\partial \varepsilon}{\partial t} + \mathbf{V}_s\nabla \cdot \varepsilon = \nabla \cdot \mathbf{V}_s = \frac{1}{(1 - n)}\frac{d_s n}{dt} \tag{5, 6}$$

Also, the use of material derivatives with respect to moving solids and substitution of (2) and (6) into (1) yields the following:

$$\nabla \cdot \mathbf{q}_r + S_w \frac{\partial \varepsilon}{\partial t} + n\frac{\partial S_w}{\partial t} - \frac{R_a}{\rho_f} = 0 \tag{7a}$$

where the water is assumed to be incompressible under pressure and temperature conditions existing in permafrost (i.e., ρ_f = constant), and the material derivates are replaced by partial ones by assuming the following:

$$\left|\frac{\partial(\ )}{\partial t}\right| >> |\mathbf{V}_s \cdot \nabla(\ )| \tag{7b}$$

Neglecting the inertial terms, the total stress tensor σ_{ij} at a point within the soil satisfies the following macroscopic equilibrium equation:

$$\frac{\partial \boldsymbol{\sigma}_{ij}}{\partial x_j} + f_i = 0, \text{ for } i = 1, 2, 3 \tag{8}$$

In Bear et al. (1984), the total stress at a macroscopic point in the porous medium domain is defined as the sum of the intergranular (or effective stress σ'_{ij}) and the pore pressure, p, in the water that, in unsaturated flow conditions, fills only part of the pore space. Following Bishop (1960), Bear et al. (1984) used a modified form of effectiveness stress for an unsaturated porous medium as follows:

$$\boldsymbol{\sigma}_{ij} = \boldsymbol{\sigma}'_{ij} - S_w p \boldsymbol{\delta}_{ij} \quad (9)$$

Separating both σ'_{ij} and p into initial steady values σ'^{o}_{ij} and p^o, and consolidation-producing incremental values of effective stress σ'^{e}_{ij} and excess pore pressure p^e, we replace (8) by the following:

$$\frac{\partial \boldsymbol{\sigma}'^{o}_{ij}}{\partial x_j} + f^o_i - \frac{\partial}{\partial x_j}(S_w p^o) = 0; \quad \frac{\partial \boldsymbol{\sigma}'^{e}_{ij}}{\partial x_j} - \frac{\partial}{\partial x_j}(S_w p^e) = 0 \quad (10, 11)$$

In writing (10) and (11), we assume that the body force remains constant.

For a thermoelastic porous medium, the stress-strain relationships are given by the Duhamel-Neumann relationships in terms of incremental strain ε^e_{kl} and incremental temperature T^3

$$\boldsymbol{\sigma}'^{e}_{ij} = C_{ijkl}\boldsymbol{\varepsilon}^e_{kl} - \beta_{ij} T^e \quad (12)$$

with material coefficients

$$C_{ijkl} = \left(\frac{\partial \boldsymbol{\sigma}'^{e}_{ij}}{\partial \boldsymbol{\varepsilon}'^{e}_{kl}}\right)\Bigg|_{T=\text{constant}}; \text{ and } \beta_{ij} = -\left(\frac{\partial \boldsymbol{\sigma}'^{e}_{ij}}{\partial T}\right)\Bigg|_{\varepsilon=\text{constant}}$$

Eq. (12) is valid under the assumptions of infinitesimal strains, $T^e/T << 1$ and dependence of free energy on the strain and temperature.

For an isotropic linear elastic body (i.e., neglecting the time-dependent creep behavior of the frozen soil), (12) reduces to the following:

$$\boldsymbol{\sigma}'^{e}_{ij} = 2G\boldsymbol{\varepsilon}^e_{ij} + (\lambda\varepsilon^e - \gamma T^e)\boldsymbol{\delta}_{ij}; \quad \varepsilon = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz} \quad (13a,b)$$

where $\gamma\boldsymbol{\delta}_{ij} = (\partial\boldsymbol{\sigma}_{ij}/\partial T)|_{\varepsilon=\text{constant}}$.

The coefficient γ is also related to the coefficient of volume thermal compression, α_T, and the coefficient of compressibility, α_p, by the following:

$$\gamma = \alpha_T/\alpha_p \quad (14)$$

For a one-dimensional (vertical) case, (13) reduces to the following:

$$\varepsilon^e_{zz} = \alpha_p \sigma'^{e}_{zz} + \alpha_T T^e \quad (15)$$

where $\alpha_p = (2G + \lambda)^{-1}$. Thus, for $\varepsilon = (\sigma', T)$, one can write the following:

$$\frac{\partial \varepsilon_{ij}}{\partial t} = \frac{\partial \varepsilon_{ij}}{\partial \boldsymbol{\sigma}'}\Bigg|_T \frac{\partial \boldsymbol{\sigma}'}{\partial T} + \frac{\partial \boldsymbol{\varepsilon}_{ij}}{\partial T}\Bigg|_p \frac{\partial T}{\partial t} = \alpha_p \frac{\partial}{\partial t}(S_w p) + \alpha_T \frac{\partial T}{\partial t} \quad (16)$$

with the assumption of a constant total stress, i.e., $d\boldsymbol{\sigma}_{ij} = 0 = d\boldsymbol{\sigma}'_{ij} - d(S_w p \delta_{ij})$, and for a one-dimensional (vertical) consolidation.

The degree of unfrozen water saturation S_w is a function of the pore-water pressure p and temperature T. Under isothermal conditions, the relation between S_w and p is given by the specific retention curve. The degree of unfrozen water saturation is a function of the pore-water pressure and temperature. Miller (1973) performed an analysis of freezing over a flat plate and concluded that experimentally determined freezing characteristic curves for a given soil at a pore-water pressure would be unique. Miller showed that, under certain conditions, the retention curve and the soil-freezing curve are strictly similar when scaled properly [e.g., Koopmans and Miller (1966)]. This applies to a saturated soil only when the curves are scaled properly, and only under the assumption that the appropriate linear combination of pressure and temperature as the independent thermodynamic variables is employed. Miller expressed the need for experimentally determined curves measured at different pore pressures. In his experiments, Miller controlled the pressure, keeping it at zero gauge, while he manipulated temperature. Under isothermal conditions, the relation between S_w and p is given by the moisture-retention curve of Fig. 1(a), and the phase composition of water in the soil under atmospheric conditions for temperatures below freezing is given in Fig. 1(b). The moisture-retention curve describes the relationship between the soil-moisture content and the pore-water pressure, i.e., $\theta_w = \theta_w(p)$. Although this relationship is subject to hysteresis, we shall assume that the process is one of drainage only. For a quantitative analysis of $\theta_w(p)$, El-Kadi (1985) reviewed four different models available in the literature. The phase diagram shows the variation of the volumetric content of unfrozen water remaining at a given temperature below freezing. This unfrozen water could be either mobile or immobile, such as adsorbed to the soil grains as a water film. There are various mathematical expressions to describe phase diagrams. The best-fit expression to the data of Jame and Norum (1980) has been supplied in a functional form by Taylor and Luthin (1978). Other equations have been provided by Dirksen and Miller (1966). Horiguchi and Miller (1983) and Phukan (1985) have supplied best-fit equations of the form $\theta_w = AT^B$, where T = temperature below freezing in °C. These equations suggest that there is no unfrozen water at T = 0°C. Numerous researchers express the phase-composition curve from the moisture-retention

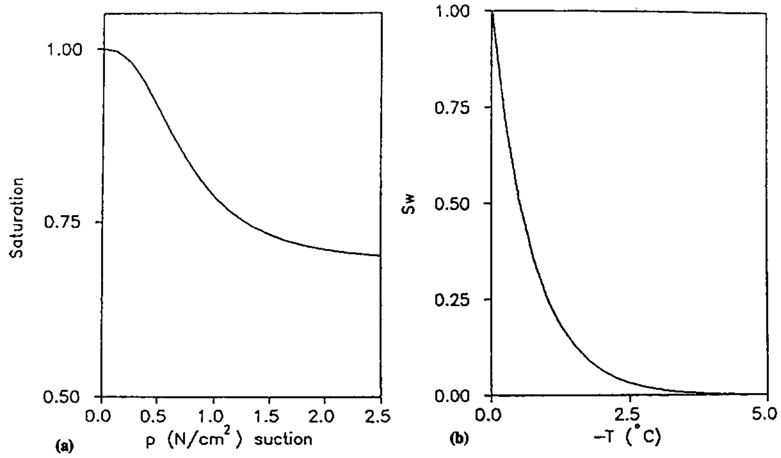

FIG. 1. Representation of Curves: (a) Specific Retention; (b) Phase Composition

curves by making use of an equilibrium relation (Clausius-Clapeyron equation) and by assuming that for a given soil matrix, the mean phase interface curvature for a draining ice-free unsaturated soil matches with that of a freezing air-free soil.

By definition, the moisture content is a product of porosity and the degree of water saturation, as follows:

$$\theta_w(p, T) = nS_w(p, T) \tag{17}$$

Then, we can write the following:

$$\frac{\partial S_w}{\partial t} = \left.\frac{\partial S_w}{\partial p}\right|_T \frac{\partial p}{\partial t} + \left.\frac{\partial S_w}{\partial T}\right|_p \frac{\partial T}{\partial t} = \xi_p \frac{\partial p}{\partial t} + \xi_p \frac{\partial p}{\partial t} + \xi_T \frac{\partial T}{\partial t} \tag{18}$$

where ξ_p and ξ_T can be obtained from (17). The link between the temperature and pressure is generally provided by the Clausius-Clapeyron equation, which is widely used in the literature [e.g., Panday and Corapcioglu (1991)]. For most problems, the temperature-based phase-composition curves are adequate. As in Miller, these are generally developed under conditions in which the unfrozen water pressure was controlled or neglected and included in the relation. Because temperature often dominates the phase composition, $\theta_w(T)$, curves obtained by neglecting the influence of pressure can be employed especially for freezing problems. The Clausius-Clapeyron equation can be expressed by the following:

$$\frac{p_w}{\rho_f} - \frac{p_i}{\rho_i} = \frac{L}{T_o}(T - T_o) \tag{19}$$

where T_o = freezing point of the pure water; L = latent heat of fusion for water; ρ_i = density of ice; and p_i = "ice pressure," which was introduced by Miller (1978) to take into account the additional stress created by soil freezing at the partially frozen front. Similar to (18), the temporal variation of ice content θ_i can be written as follows:

$$\frac{\partial \theta_i}{\partial t} = \left.\frac{\partial \theta_i}{\partial p}\right|_T \frac{\partial p}{\partial t} + \left.\frac{\partial \theta_i}{\partial T}\right|_p \frac{\partial T}{\partial t} = \xi_{ip} \frac{\partial p}{\partial t} + \xi_{iT} \frac{\partial T}{\partial t} \tag{20}$$

Although the phase-composition curve experiences hysterises during the freezing and thawing cycles (Koopmans and Miller 1966), in this study, as noted earlier, the effect of hysterises has been neglected. We realize that retention curves and phase-composition curves are obtained under equilibrium conditions. The use of such equilibrium curves in studies involving transient-flow problems in unsaturated soils, especially in slow-varying problems like thaw-consolidation, is a valid procedure.

The unfrozen water flow equation can finally be completed by expanding and substituting (16) for $\partial\varepsilon/\partial t$, and using (18) for $\partial S_w/\partial t$ in (7)

$$\nabla \cdot \mathbf{q}_r + (S_w p \alpha_p \xi_p + \alpha_p S_w^2 + \eta \xi_p) \frac{\partial p}{\partial t} + (S_w p \alpha_p \xi_T + S_w \alpha_T + \eta \xi_T) \frac{\partial T}{\partial t} - \frac{R_a}{\rho_f} = 0 \tag{21}$$

where $\mathbf{q}_r$ can be expressed using (2).

Conservation of Mass Equation for Ice

The conservation of mass equation for the melting ice in a porous medium is given by the following:

$$\frac{\partial}{\partial t}(\rho_i\theta_i) + \nabla\cdot(\rho_i\theta_i\mathbf{V}_s) + R_a = 0 \tag{22}$$

where θ_i = ice content. It is assumed here that during consolidation due to melting, ice moves with the same velocity as the solid particles. This is especially true when ice particles are at the very close proximity to grain surfaces separated by water films. Therefore, an ice particle in a pore moves with the solid particles circumscribing the pore. This assumption is particularly true when the frozen soil thaws. In freezing soils, soil particles might be separated due to heave. With the assumption that the density of ice, ρ_i, remains constant, and by employing the material derivative concept, one obtains the following:

$$\frac{d_s\theta_i}{dt} + \theta_i\nabla\cdot\mathbf{V}_s + \frac{R_a}{\rho_i} = 0 \tag{23}$$

By substituting $\nabla\cdot\mathbf{V}_s$ from (6), replacing material derivatives by partial ones, and inserting (16) and (18) for $\partial\varepsilon/\partial t$ and $\partial S_w/\partial t$ into (23), we obtain the following:

$$\frac{\partial\theta_i}{\partial t} + (\theta_i p\alpha_p\xi_p + \theta_i\alpha_p S_w)\frac{\partial p}{\partial t} + [\theta_i(p\alpha_p\xi_T + \alpha_T)]\frac{\partial T}{\partial t} + \frac{R_a}{\rho_i} = 0 \tag{24}$$

where $\partial\theta_i/\partial t$ can be expressed in terms of p and T by (20).

Conservation of Energy Equation

The macroscopic energy-conservation equation for a saturated porous medium was developed by Bear and Corapcioglu (1981), starting from microscopic considerations and deriving the macroscopic one by averaging the former over a representative elementary volume of the porous medium. It is assumed that the thermal resistance between water and soil particles is small; therefore local water and matrix temperatures are equal. The modification of Bear and Corapcioglu's (1981) equation to incorporate the phase change due to melting in an unsaturated thawing deformable porous medium would give the following:

$$\frac{\partial}{\partial t}[(\rho C)_m T] - LR_a + \nabla\cdot\{[\theta_w\rho_f C_V\mathbf{V}_f + \theta_i\rho_i C_i\mathbf{V}_s + (1-n)\rho_s C_r\mathbf{V}_s]T\}$$

$$- \nabla\cdot[\Lambda_m\nabla T] - p\nabla\cdot(\theta_w\mathbf{V}_f) + [\theta_i + (1-n)]T\gamma\frac{\partial\varepsilon}{\partial t} = 0 \tag{25}$$

The heat capacity per unit volume of the porous medium $(\rho C)_m$, and the coefficient of thermal conductivity of the porous medium as a whole Λ_m, can be written as follows:

$$(\rho C)_m = \theta_w C_V\rho_f + \theta_i C_i\rho_i + (1-n)\rho_s C_r; \quad \mathbf{\Lambda}_m = \theta_w\lambda_f + \theta_i\lambda_i + (1-n)\lambda_s \tag{26, 27}$$

A review shows that different forms of expressions for $(\rho C)_m$ and $\mathbf{\Lambda}_m$ are available in the literature [e.g., Farouki (1981), and Lachenbruch et al. (1982)]. In (26) and (27), we neglect the heat capacity and thermal conductivity of the air phase.

The first term of (25) is the rate of change of the total heat content. The second term represents the rate of heat production due to melting. The third term represents the transfer of sensible heat by convection and deformation. The fourth term contributes to pure heat conduction, and the last two terms express the source of heat due to internal energy increase per unit volume of porous medium by viscous dissipation and compression, respectively. The first term of (25) can be expanded using (26) as follows:

$$\frac{\partial}{\partial t}[(\rho C)_m T] = (\rho C)_m\frac{\partial T}{\partial t} + T\frac{\partial}{\partial t}[(\rho C)_m] \tag{28a}$$

$$\frac{\partial}{\partial t}[(\rho C)_m T] = (\rho C)_m\frac{\partial T}{\partial t} + T\frac{\partial}{\partial t}[\theta_w C_V\rho_f + \theta_i C_i\rho_i + (1-n)C_r\rho_s] \tag{28b}$$

$$\frac{\partial}{\partial t}[(\rho C)_m T] = (\rho C)_m\frac{\partial T}{\partial t} + TC_V\frac{\partial}{\partial t}(\theta_w\rho_f) + TC_i\frac{\partial}{\partial t}(\rho_i\theta_i) + TC_r\rho_s\frac{\partial}{\partial t}(1-n) \tag{28c}$$

Substitution of (1) and (22) into (28) yields the following:

$$\frac{\partial}{\partial t}[(\rho C)_m T] = (\rho C)_m\frac{\partial T}{\partial t} + TC_V[-\rho_f\nabla\cdot\mathbf{q} + R_a] + TC_i(-R_a - \rho_i\nabla\cdot\theta_i\mathbf{V}_s) + TC_r\rho_s\frac{\partial}{\partial t}(1-n) \tag{29}$$

and replacement of $\mathbf{q}$ by (2) gives the following:

$$\frac{\partial}{\partial t}[(\rho C)_m T] = (\rho C)_m \frac{\partial T}{\partial t} - TC_{\text{v}}\rho_f \nabla\cdot\mathbf{q}_r + TR_a(C_{\text{v}} - C_i) - C_{\text{v}}\rho_f T\nabla\cdot\theta_w\mathbf{V}_s$$

$$- C_i T\rho_i\nabla\cdot\theta_i\mathbf{V}_s - TC_s\rho_s\nabla\cdot(1 - n)\mathbf{V}_s \quad (30)$$

The last three terms of (30) can be manipulated such that (30) can be written in the following form:

$$\frac{\partial}{\partial t}[(\rho C)_m T] = (\rho C)_m \frac{\partial T}{\partial t} - TC\rho_f\nabla\cdot\mathbf{q}_r + TR_a(C_{\text{v}} - C_i) - T(\rho C)_m\nabla\cdot\mathbf{V}_s - T\mathbf{V}_s\cdot\nabla(\rho C)_m \quad (31)$$

The third term of (25) can be expressed using (26), as follows:

$$\nabla\cdot\{[\theta_w\rho_f C_{\text{v}}\mathbf{V}_f + \theta_i\rho_i C_i\mathbf{V}_s + (1 - n)\rho_s C_s\mathbf{V}_s]T\} = \nabla\cdot\{[\theta_w\rho_f C_{\text{v}}(\mathbf{V}_f - \mathbf{V}_s) + (\rho C)_m\mathbf{V}_s]T\} \quad (32)$$

and the whole term within the divergence operator can be expanded. Furthermore, the fifth term of (25) can also be expanded employing (1) and (2) as follows:

$$-p\nabla\cdot\theta_w\mathbf{V}_f = -p\nabla\cdot\mathbf{q} - p\left(\frac{\partial\theta_w}{\partial t} - \frac{R_a}{\rho_f}\right) \quad (33)$$

Rewriting (25) with (31), (32), and (33), replacing those respective terms, and again neglecting the convective part of d_sT/dt, we obtain the following:

$$\frac{\partial T}{\partial t} + \mathbf{V}_s\cdot\nabla T = \frac{d_sT}{dt} \approx \frac{\partial T}{\partial t} \quad (34a)$$

$$(\rho C)_m\frac{\partial T}{\partial t} + R_a[T(C_{\text{v}} - C_i) - L - (-p)/\rho_f] - \nabla\cdot[\mathbf{\Lambda}_m\nabla T] + C_{\text{v}}\rho_f\mathbf{q}_r\nabla\cdot T$$

$$+ p\frac{\partial}{\partial t}(nS_w) + [\theta_i + (1 - n)]T_\gamma\frac{\partial\varepsilon}{\partial t} = 0 \quad (34b)$$

The last two terms of (34) can be expanded using (18), (16), and (6), giving the final form as follows:

$$\{(\rho C)_m + pn\xi_T + (p\alpha_p\xi_T + \alpha_T)[(\theta_i + 1 - n)T_\gamma + (1 - n)pS_w]\}\frac{\partial T}{\partial t}$$

$$+ \{pn\xi_p + (p\xi_p + S_w)[(1 - n)T\gamma\alpha_p + (1 - n)S_w p\alpha_p]\}\frac{\partial p}{\partial t}$$

$$+ C_{\text{v}}\rho_f q_r\nabla\cdot(\mathbf{\Lambda}_m\nabla T) + R_a[T(C_{\text{v}} - C_i) - L - p/\rho_f] = 0 \quad (35)$$

Eq. (35) is the final form of heat transport equation to be employed in this study. Its fundamental dependent variables are unfrozen water pressure p and soil temperature T.

MATERIAL PARAMETERS

To obtain a solution to the system of equations developed previously, one needs the hydraulic and thermal parameters of the soil system. Properties like α_p, ρ_i, ρ_f, ρ_s, C_i, C_s, λ_i, λ_f, λ_s, and the latent heat of fusion, L, can be assumed to be constraints within the pressure and temperature limits under consideration. However, parameters like the hydraulic conductivity and the degree of saturation and its variation with pressure and temperature ξ_p and ξ_T are not only pressure dependent, but also greatly temperature dependent for temperatures below freezing. Various types of functional expressions have been reported in the literature. Typical retention curves and phase diagrams as illustrated by Figs. 1(a) and 1(b), respectively.

The hydraulic conductivity of an unsaturated porous medium is a function of the liquid-water content θ_w, and can be expressed by the Averjanov-Irmay function (El-Kadi 1985), as follows:

$$K = K_s S_e^N \quad (36)$$

where K_s = intrinsic permeability and the effective saturation S_e is defined by $S_e = (\theta_w - \theta_r)/(\theta_o - \theta_r)$. Alternative forms of (36), such as exponential functions, have been proposed in the literature (Farouki 1981).

A study by Jame and Norum (1980), assuming the moisture diffusivity as a function of the liquid-water content only, indicated an excessive accumulation of water behind the freezing front and a sharp decrease in water content in the unfrozen region towards the frozen face. Conclusions drawn instigated them to introduce an impedance factor, assumed to be a function of the total ice content. Taylor and Luthin (1978) discovered that the wide disparity in their simulations was eliminated by introducing such an impedance factor, I, in the freezing zone. Furthermore, changing viscosity effects have been mentioned by Iwata and Tabucki (1988) as contributing to the use of an impedance factor. The relationship for this impedance factor was suggested as follows:

$$I = 10^{10\theta_i} \tag{37}$$

and the hydraulic conductivity in the frozen zone is calculated by the following:

$$K_{\text{frozen}} = K_{\text{unfrozen}}/I \tag{38}$$

The compressibility coefficient due to thermal changes, α_T, is a parameter not readily found in literature. However, it can be determined from the thaw-settlement parameter, A_o. For complete thawing of a fully frozen, fully saturated medium, the thaw-settlement parameter A_o as defined by Sykes et al. (1974b) represents the thaw settlement due to the volume change of ice on melting and release of pore water. Watson et al. (1973) have performed experiments to determine the value of A_o. By definition, for a saturated medium, $\theta_w = n$. Thus, initially, if all of the pore spaces are ice-filled, i.e., $\theta_i = n_o$, then after total melting, i.e., $\theta_i = 0$, incremental change in ice content is the following:

$$\Delta\theta_i = n_o - 0 = n_o \tag{39}$$

In this case, $\partial\varepsilon|_p = A_o$, where A_o = thaw-settlement parameter based on the definition of Sykes et al. (1974b). Sykes et al. (1974b) showed that for a fully saturated frozen soil

$$A_o = \left.\frac{\Delta e}{1 + e}\right|_p \tag{40}$$

or

$$\left.\frac{\Delta e}{1 + e}\right|_p = \Delta n| \equiv \partial\varepsilon|_p \tag{41}$$

Now, assuming a linear relationship for a change of ice content, $\partial\theta_i$, the strain would then be the following:

$$\partial\varepsilon|_p = A_o \frac{\partial\theta_i}{n_o} \tag{42}$$

In other words, (42) assumes that the strain at any time, due to melting at constant pressure, is proportional to A_o, and the proportionality constant is the ratio of change of ice content to initial porosity. This ratio is equal to unity when the fully saturated frozen soil totally melts. Now, in (16), α_T has been defined such that

$$\partial\varepsilon|_p = \alpha_T \partial T \tag{43}$$

Comparison of (42) and (43) gives the following:

$$\alpha_T = \frac{A_o}{n_o}\frac{\partial\theta_i}{\partial T} = \frac{A_o}{n_o}\xi_{iT} \tag{44}$$

Finally, the thermal liquid diffusivity is also given as a function of the moisture content, and curves for the relationship have been supplied [e.g., Farouki (1981), p. 49] after Higashi (1958). Least-squares fit expressions can be obtained from such curves for the purpose of numerical solutions.

Approximation for Column Experiment

The model developed earlier provides a general formulation to analyze the water flow and consolidation of an unsaturated partially frozen permafrost. A fully coupled solution of governing equations might require very large computer facilities and highly sophisticated algorithms. Panday and Corapcioglu (1995) provides the numerical solution and sensitivity analysis of the formulation obtained in this paper. Therefore, to provide the set of equations to be solved in Panday and Corapcioglu (1995), the general formulation will be expressed in one-dimensional form to simulate a frozen column. Hence, the ice-mass conservation equation [(24)] can be written as follows:

$$-R_a = [\rho_i\xi_{ip} + \rho_i\theta_i\alpha_p(p\xi_p + S_w)]\frac{\partial p}{\partial t} + [\rho_i\xi_{iT} + \rho_i\theta_i(p\alpha_p\xi_T + \alpha_T)]\frac{\partial T}{\partial t} \tag{45}$$

The flow equation (21) with $\mathbf{q}_r$ represented as in (2), and coupled by the R_s term to (45), can be written as follows:

$$\frac{\partial}{\partial z}\left[-K\left(\frac{1}{\rho_f g}\frac{\partial p}{\partial z}-1\right)-\mathbf{D}_{MT}\frac{\partial T}{\partial z}\right]+\left[S_w p\alpha_p\xi_p+\alpha_p S_w^2+n\xi_p+\left(\frac{\rho_i}{\rho_f}\right)\xi_{ip}\right.$$

$$\left.+\left(\frac{\rho_i}{\rho_f}\right)\theta_i\alpha_p(p\xi_p+S_w)\right]\frac{\partial p}{\partial t}+\left[S_w p\alpha_p\xi_T+S_w\alpha_T+n\xi_T\right.$$

$$\left.+\left(\frac{\rho_i}{\rho_f}\right)\xi_{iT}+\left(\frac{\rho_i}{\rho_f}\right)\theta_i(p\alpha_p\xi_f+\alpha_T)\right]\frac{\partial T}{\partial t}=0 \tag{46}$$

and the energy equation (35) in its one-dimensional form coupled with (45) is given by the following:

$$\left\{(\rho C)_m+pn\xi_T+(p\alpha_p\xi+\alpha_T)[(\theta_i+1-n)t\gamma+(1-n)pS_w]-[\rho_i\xi_{iT}+\rho_i\theta_i(p\alpha_p\xi_T+\alpha_T)]\right.$$

$$\left.\cdot\left[T(C_\nabla-C_i)-L-\frac{p}{\rho_f}\right]\right\}\frac{\partial T}{\partial t}+\left\{pn\xi_p+(p\xi_p+S_w)[(1-n)T\gamma\alpha_p+(1-n)S_w p\alpha_p]\right.$$

$$\left.-[\rho_i\xi_{ip}+\rho_i\theta_i\alpha_p(p\xi_p+S_w)]\left[T(C_\nabla-C_i)-L-\frac{p}{\rho_f}\right]\right\}\frac{\partial p}{\partial t}+C_\nabla\rho_f q_r\frac{\partial T}{\partial z}-\frac{\partial}{\partial z}\left(\mathbf{\Lambda}_m\frac{\partial T}{\partial z}\right)=0 \tag{47}$$

In (46) and (47) provide a starting point for a numerical solution to the basic variables, p and T, of a vertical one-dimensional soil column. As consolidation of the soil column proceeds, the porosity values can be updated by combining (6) and (16) and neglecting the convective terms as follows:

$$\frac{1}{(1-n)}\frac{dn}{dt}=\alpha_p(S_w+p\xi_p)\frac{\partial p}{\partial t}+\alpha_T\frac{\partial T}{\partial t} \tag{48}$$

The settlements of the column can be calculated by considering the changes in porosity. If the porosity changes by Δn, the column length will change by an amount, $\delta\Delta z=\Delta n\Delta z_o$, because the solids themselves are assumed to be incompressible. Also, Δz_o = initial length of the column segment, and $\delta\Delta z$ = change in this length. Thus, under the assumption of one-dimensional settlement, the new length of that element will be the following:

$$\Delta z_{\text{new}}=\Delta z_o+\delta\Delta z=(1+n-n_o)\Delta z_o \tag{49}$$

The new total length of the column will be the sum of the new length segments.

Summary and Conclusions

A mathematical model describing the conditions existing in a partially saturated thawing soil column has been presented. The model includes conservation of mass equations for liquid water, frozen water (ice), soil grains, conservation of energy equation, and fundamental definitions of stress and strain. The soil air is assumed to be at the atmospheric pressures. Phase changes other than freezing and melting have been assumed to be negligible. The governing equations have been simplified to obtain the final one-dimensional form suitable for solution by numerical techniques. Assumptions valid for processes involved in thawing soils have been incorporated to present governing equations in a form amenable to numerical solution. The water, soil grains, and ice have been assumed to be incompressible. Specific retention and phase-composition curves, which are obtained under equilibrium conditions, have been utilized, and their involvement in slow-varying problems such as this one is justified. The material coefficients of the model involved in the solution and their relation to the conditions existing within the soil are also presented briefly. During consolidation, it is assumed that there is no relative movement between the soil and ice particles, and they they compress as one, moving with the same velocity. Also, the assumption that replaces material derivatives by partial ones requires $V_s\cdot\nabla(\ ) << \partial(\ )/\partial t$. By this, we imply that the local part of the total derivative is at least an order of magnitude larger than the convective part. In the case of thaw consolidation, we deal with small spatial gradients and/or negligibly small velocities due to the nature of the phenomenon.

Panday and Corapcioglu (1995), the one-dimensional mathematical model as presented is numerically solved for a soil column, and the sensitivity of the model to various parameters are investigated.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A_o = thaw settlement parameter;
C_i = gravimetric heat capacity of ice (J/kg/°C);
C_s = gravimetric heat capacity of soil solids (J/kg/°C);
C_w = gravimetric heat capacity of water (J/kg/°C);
$\mathbf{D}_{MT}$ = thermal liquid diffusivity ($cm^2 s^{-1} °C^{-1}$);
$d_s(\ )/dt$ = total derivative with respect to moving solid;
G = shear modulus (N/m^2);
g = acceleration due to gravity (m/s^2);
$\mathbf{K}$ = hydraulic conductivity (m/s);
L = latent heat of fusion (J/kg);
n = porosity of porous medium;
n_o = initial porosity;
p = pore-water pressure (N/m^2);
$\mathbf{q}$ = specific discharge of water (m/s);
$\mathbf{q}_r$ = specific discharge of water relative to moving solid (m/s);
R_a = rate of melting of ice mass ($kg/m^3/s$);
S_w = degree of saturation;
T = temperature (°C);
t = time (s);
$\mathbf{U}$ = solids displacement (m);
$\mathbf{V}_f$ = velocity of flowing water (m/s);
$\mathbf{V}_s$ = velocity of solid particles (m/s);
z = vertical coordinate (m);
α_p = compressibility of soil (m^2/N);
α_T = thermal compressibility of soil (m/°C);
γ = $\partial\sigma'/\partial T|_\epsilon = \partial t/\partial p$;
Δt = time increment (s);
Δz_o = initial length of column segment (m);
$\boldsymbol{\delta}_{ij}$ = Kronecker delta;
ε = strain;
θ_i = volumetric ice content;

θ_w = volumetric water content;
$\mathbf{\Lambda}_m$ = coefficient of thermal conductivity of porous medium (J/m/s/°C);
λ = Lame's constant (N/m^2);
λ_f = coefficient of thermal conductivity of water (J/m/s/°C);
λ_i = coefficient of thermal conductivity of ice (J/m/s/°C);
λ_s = coefficient of thermal conductivity of soil solids (J/m/s/°C);
ξ_{ip} = change of ice content with respect to change in pore pressure;
ξ_{iT} = change of ice content with respect to change in temperature;
ξ_p = change of water content with respect to change in pore pressure;
ξ_T = change of water content with respect to change in temperature;
ρ_f = density of water (kg/m^3);
ρ_i = density of ice (kg/m^3);
ρ_s = density of soil (kg/m^3);
$(\rho C)_m$ = heat capacity of porous medium (J/kg/°C);
$\boldsymbol{\sigma}$ = total stress (N/m^2); and
$\boldsymbol{\sigma}'$ = effective stress (N/m^2).